Central Banks Balance Sheet Policies Without Rational Expectations

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Central Banks Balance Sheet Policies

Examples

- QE (long-term public and private assets purchases)
- FX interventions

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"The problem with QE is that it works in practice, but it does not work in theory"

Ben Bernanke (2014)

Empirics

QE

- Gagnon-Raskin-Remache-Sack (2011), Krishnamurthy-Vissing-Jorgensen (2011), Hancock-Passmore (2011), Di Maggio-Kermani-Palmer (2016), Chakraborty-Goldstein-MacKinlay (2016), Fieldhouse-Mertens-Ravn (2018)

FX interventions

- Dominguez-Frankel (1990, 1993), Dominguez (1990, 2006), Catte-Galli-Rebecchini (1994), Kearns-Rigobon (2005), Blanchard-Adler-de Carvalho (2014), Fratzscher-Gloede-Menkhoff-Sarno-Stohr (2015)

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Theory

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1. people can freely trade targeted assets
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3. people correctly predict future effects of policies

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2. Signaling channel (asymmetric info or limited commitment)

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This paper: bounded rationality channel

- Beliefs about future deviate from rational expectations
- Agents do not fully understand future effects of the policies

Deviations from Rational Expectations

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Induction

- *Idea*: beliefs about future are refined over time through a process of induction from observed outcomes.
- *Econometric learning*: Evans-Honkapohja; Shleifer; etc.

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Eduction

- *Idea*: agents understand the model and form expectations from it about future outcomes through the process of reflection
- ***Level-k thinking***

Level-k Thinking

Game Theory

- Stahl-Wilson (1994,1995); Nagel (1995); Crawford (2013)
- Idea: Agents know the game; rationally respond to beliefs; form beliefs about opponents actions recursively
- Result: Level-k thinking better approximates experimental results in strategic games (more so in new games)

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Macro

- Evans-Ramey; GarciaSchmidt-Woodford; Farhi-Werning
- Idea: Agents know the model; optimize; form expectations about future endogenous variables recursively
- Result: Level-k thinking dampens changes in expectations about future endogenous variables after new policies

A Simple Model

A Simple Model

OLG households solve

$$\max_{x_{t+1}, s_{t+1}, c_{t+1}} \widetilde{\mathbb{E}}_t \frac{e^{-\gamma c_{t+1}}}{-\gamma}$$

$$\text{s.t.:} \quad \frac{s_t}{R} + q_t x_t \leq w$$

$$c_{t+1} + T_{t+1} \leq (r^x + \epsilon^x_{t+1} + q_{t+1}) x_t + s_t$$

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"Assumption"

beliefs about future endogenous variables

$$\widetilde{q}_{t+s} = \alpha_{q,t+s} + \beta_{q,t+s} \epsilon^x_{t+s}$$

$$\widetilde{T}_{t+s} = \alpha_{T,t+s} + \beta_{T,t+s} \epsilon^x_{t+s}$$

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Risky assets demand

$$x(q_t; \{\widetilde{q}_{t+s}, \widetilde{T}_{t+s}\}) = \frac{r^x + \widetilde{\mathbb{E}}_t q_{t+1} - q_t R}{\gamma \sigma^2_x} + \beta_{T,t+1}$$

Government

- Central bank announces path of purchases $\{X_{t+1}, B_{t+1}\}$

- Treasury collects policy profits and turns into taxes

$$T_t = \begin{cases} q_0 X_1 - \frac{B_1}{R}, & t = 0 \\ q_t X_{t+1} + B_t - \frac{B_{t+1}}{R} - \left(r^x + \epsilon_t^x + q_t\right) X_t, & t \geq 1 \end{cases}$$

Temporary Equilibrium (TE)

Idea: TE takes as given a sequence of beliefs and imposes that markets clear in every period (Hicks; Lindahl; Grandmont)

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Definition For $\{\widetilde{T}_t, \widetilde{q}_t\}$, a TE is $\{X_{t+1}, B_{t+1}, T_t; q_t; s_{t+1}, x_{t+1}, c_t\}$

$$\frac{r^x + \widetilde{\mathbb{E}}_t q_{t+1} - q_t R}{\gamma \sigma_x^2} + \beta_{T,t+1} = \overline{X} - X_{t+1}$$

$\{c_t, s_{t+1}\}$ are optimal, and

$$T_t = q_t X_{t+1} - \frac{B_{t+1}}{R} + B_t - (r^x + \epsilon_t + q_t) X_t$$

REE

Level-k thinking Belief Formation

Status quo $\{\widetilde{q}_{t+s}, \widetilde{T}_{t+s}\} = \{q^{REE}, 0\}$ (REE before intervention)

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Level-1 Thinking

$$\left.\begin{aligned} & x(q_t^1; \{q^{REE}, 0\}) = \overline{X} - X_{t+1} \\ & T_t^1 = q_t^1 X_{t+1} - \frac{B_{t+1}}{R} + B_t - \left(r_t^x + q_t^1\right) X_t \end{aligned}\right\} \Rightarrow \{T_t^1, q_t^1\}$$

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Level-2 Thinking

$$\left.\begin{aligned} & x\left(q_t^2; \{q_{t+s}^1, T_{t+s}^1\}\right) = \overline{X} - X_{t+1} \\ & T_t^2 = q_t^2 X_{t+1} - \frac{B_{t+1}}{R} + B_t - \left(r_t^x + q_t^2\right) X_t \end{aligned}\right\} \Rightarrow \{T_t^2, q_t^2\}$$

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Level-k Thinking
$$\{q_t^k, T_t^k\} = \Psi(\{q_t^{k-1}, T_t^{k-1}\}; \{X_{t+1}, B_{t+1}\})$$

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REE $\{q_t, T_t\} = \Psi(\{q_t, T_t\}; \{X_{t+1}, B_{t+1}\})$

Level-k thinking Belief Formation

$$q_t^k = \begin{cases} \dfrac{r^x + q^{REE} - \gamma \sigma_x^2 \left(\overline{X} - X_{t+1} \right)}{R}, & k = 1 \\ \dfrac{r^x + q_{t+1}^{k-1} - \gamma \sigma_x^2 \overline{X}}{R}, & k > 1 \end{cases}$$

Forward Iteration

The Euler equation in REE

$$q_t = \frac{r^x + \mathbb{E}_t q_{t+1} - \gamma \sigma_x^2 \overline{X}}{R}$$

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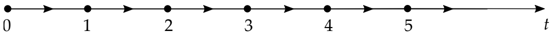
Forward Iteration



No-bubble solution

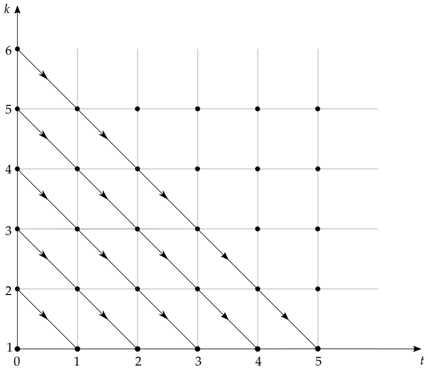
$$q_t = \frac{r^x - \gamma \sigma^2 \overline{X}}{R - 1} \equiv q^{REE}$$

Diagonal Iteration

$$q_t^k = \frac{r^x + q_{t+1}^{k-1} - \gamma\sigma^2\overline{X}}{R},\ q_{t+k-1}^1 = \frac{r^x + q^{REE} - \gamma\sigma_x^2\left(\overline{X} - X_{t+k}\right)}{R}$$

Diagonal Iteration

$$q_t^k = \frac{r^x + q_{t+1}^{k-1} - \gamma\sigma^2\overline{X}}{R},\ q_{t+k-1}^1 = \frac{r^x + q^{REE} - \gamma\sigma_x^2\left(\overline{X} - X_{t+k}\right)}{R}$$



Endogenous discounting

Reflective Equilibrium

Idea: agents form beliefs according to level-k thinking, the economy is populated by agents with different k with pdf $f(k)$ (Woodford;GarciaSchmidt-Woodford)

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In case of exponential $f(k)$ with average $\overline{k}$

$$q_t = q^{REE} + \gamma\sigma_x^2 \frac{\sum_{k=1}^{\infty} \left(\frac{\overline{k}-1}{\overline{k}}\right)^{k-1} \frac{X_{t+k}}{R^k}}{\overline{k}}$$

Higher $\overline{k}$

1. reduces the direct effect of interventions
2. makes the price react more to expected future interventions

Numerical Illustration

Extensions

1. Foreign exchange interventions (+nominal variables) Details

2. Long-term public bonds purchases (+nominal variables)
   - Model with level-1 thinkers resembles Vayanos-Vila (2009)

3. Learning to play equilibrium Details
   - Existing policy effect disappears over time
   - New policies are less effective

4. Limited participation + level-k thinking
   - Negative interaction

5. Presence of rational expectations agents Details
   - Does not change qualitative results
   - Can amplify effects of level-k thinking

Testable Predictions

Forecast errors

- Agents make predictable forecast errors
- The errors can differentiate the model from other theories (segmented markets, signaling channel)

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Forecast errors in the model

$$\text{Individual: } u^k_{t,t+s} \equiv q_{t+s} - \widetilde{\mathbb{E}}^k_t q_{t+s}$$

$$\text{Average: } \bar{u}_{t,t+s} \equiv \sum_{k=1}^{\infty} f(k) u^k_{t,t+s} = \mu^s \frac{\gamma \sigma_x^2 X_{t+1}}{\bar{k}[(R-\mu)\bar{k} + \mu]}$$

Empirics

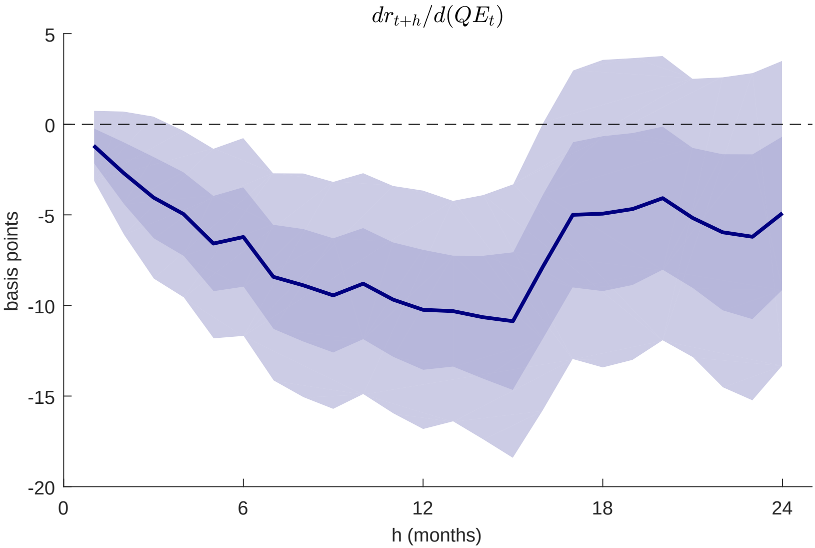
Fieldhouse-Mertens-Ravn (2018, QJE)

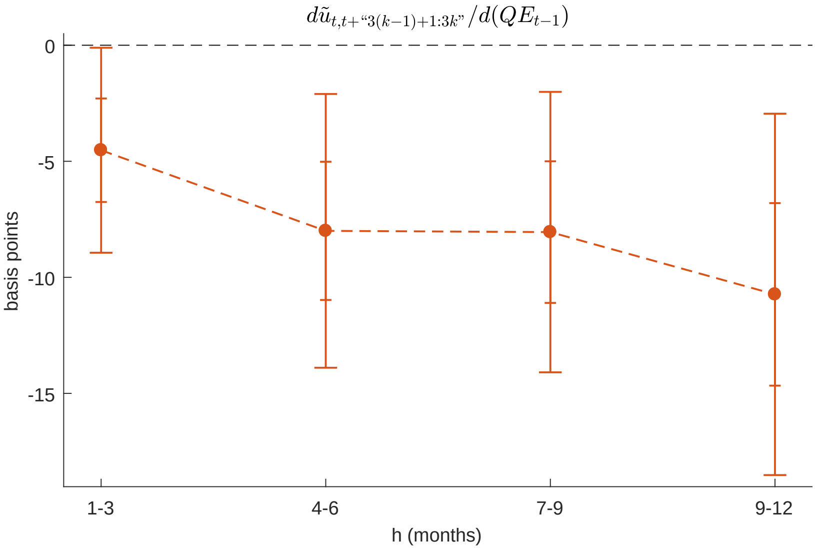
- Monthly data on GSEs mortgage purchases: 1967-2006
- "Unexpected exogenous" purchases narrative identification
- Result: mortgage yield reacts significantly to interventions

Forecast errors

- Blue Chip conventional mortgage rate forecasts: 1982-2006
- Project median forecast errors on "exogenous" purchases

Details


$dr_{t+h}/d(QE_t)$
5
0
-5
-10
-15
-20
basis points
0
6
12
18
24
h (months)


$d\tilde{u}_{t,t+``3(k-1)+1:3k"}/d(QE_{t-1})$
0
-5
-10
-15
basis points
1-3
4-6
7-9
9-12
h (months)

Conclusion

1. Bounded rationality channel of balance sheet policies
    - Level-k thinking belief formation

2. Testable predictions
    - Forecast errors respond to interventions
    - Evidence from mortgage rate forecasts errors

Rational Expectations Equilibrium

Definition: REE is a TE such that

$$\widetilde{T}_t = T_t,\ \widetilde{q}_t = q_t$$

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Specifically

$$\underbrace{\alpha_{T,t} + \beta_{T,t}\epsilon_t}_{\text{tax beliefs } \widetilde{T}_t} \overset{\text{REE}}{=} \underbrace{q_t X_{t+1} - \frac{B_{t+1}}{R} + B_t - X_t(q_t + r^x) - X_t\epsilon_t}_{\text{realized taxes } T_t}$$

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Risky assets market in t

$$\frac{r^x + \mathbb{E}_t q_{t+1} - q_t R}{\gamma\sigma_x^2} + \beta_{T,t+1} = \overline{X} - X_{t+1}$$

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Balance sheet policy does not affect price q_t in REE!

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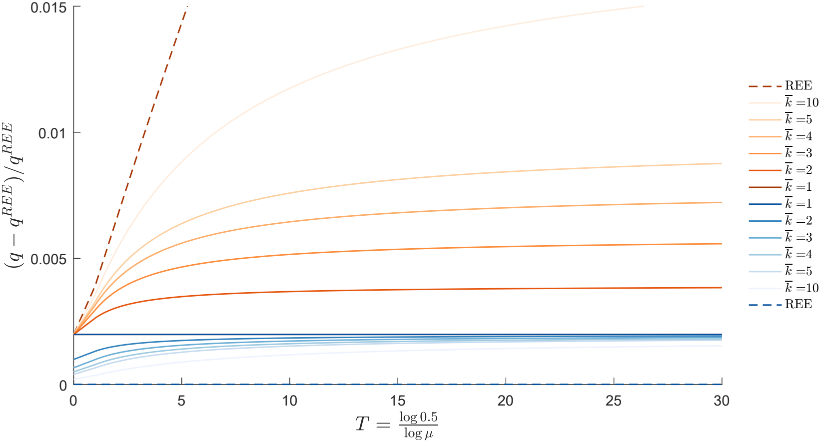
Numerical Illustration



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Open Economy Model

New elements

- **2 countries**: Home (size ω), Foreign (size $1-\omega$)
- **Additional assets**: riskless nominal bonds (in each country), money (in each country)
- **Goods**: single traded good (LOOP holds)
- **Risk**: money supply (=inflation risk)
- **Monetary policy**: $\log M_{t+1} = \log \overline{M} + \epsilon_t^h$ (home)
- **FX intervention**: $\{-B_{t+1}^h, B_{t+1}^f\}$ (home)

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Solution

Money markets

$$m_t - p_t = -vi_t$$
$$m_t^* - p_t^* = -vi_t^*$$

Bond markets

$$\text{Home:} \quad \omega b_{H,t+1} + (1-\omega) b^*_{H,t+1} = -B^h_{t+1}$$
$$\text{Foreign:} \quad \omega b_{F,t+1} + (1-\omega) b^*_{F,t+1} = B^* - B^f_{t+1}$$

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Nominal Exchange Rate

Rational expectations equilibrium

$$e_t = \frac{v}{1+v}\mathbb{E}_t e_{t+1} + \frac{1}{1+v}(m_t - m_t^*) \;\Rightarrow\; e_t^{REE}$$

Reflective equilibrium (summing over different level-k's)

$$e_t = e_t^{REE} + \frac{\gamma}{(1+v)^2}\sum_{k=1}^{\infty} f(k)\left(\frac{v}{1+v}\right)^{k+2}\left[\sigma_f^2 B_{t+k}^f + \sigma_h^2(-B_{t+k}^h)\right]$$

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Unraveling

Assumption: agents become more sophisticated over time

$$f_t\left(k\right) = \begin{cases} f(k-ht), & k \geq 1+ht, \\ 0, & k < 1+ht, \end{cases}$$

When asset purchases follow $X_{t+1} = \mu^t X_1$

$$q_t - q^{REE} = \frac{\gamma\sigma_x^2}{\omega} \cdot \frac{1}{\frac{R-\mu}{1-\lambda} + \mu} X_1 \left(\frac{\mu^{1+h}}{R^h}\right)^t$$

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REEE Agents

Assumption

- ϕ agents form expectations rationally
- $(1-\phi)$ agents form expectations using level-k thinking

Risky asset price

$$q_t = q^{REE} + (1-\phi)\,\gamma\sigma_x^2 \sum_{k=1}^{\infty} \frac{f(k)}{R^k} \sum_{s=0}^{\infty} \left(\frac{\phi}{R}\right)^s X_{t+s+k}$$

Risky asset price without REE agents

$$q_t = q^{REE} + \gamma\sigma_x^2 \sum_{k=1}^{\infty} \frac{f(k)}{R^k} X_{t+k}$$

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Fieldhouse-Mertens-Ravn (2018)

1st stage

$$\frac{\sum_{j=0}^{h} p_{t+j}}{X_t} = \alpha_h + \gamma_h \frac{m_t}{X_t} + \varphi_h(L) Z_{t-1} + u_{t+h}$$

- p_t – policy action (agency commitments)
- m_t is the noncyclical narrative policy indicator,
- X_t - a deterministic trend in real personal income

2nd stage

$$y_{t+h} - y_{t-1} = \alpha_h + \gamma_h \underbrace{\left(\frac{12}{8} \times \frac{\sum_{j=0}^{h} p_{t+j}}{\widetilde{X}_t} \right)}_{\equiv QE_t} + \varphi_h (L) Z_{t-1} + u_{t+h}$$

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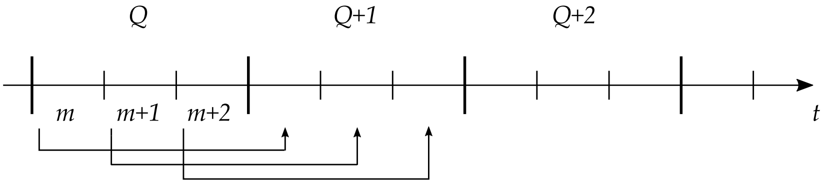
Forecast Errors

Forecasts $\widetilde{\mathbb{E}}_t r_{t+h}$



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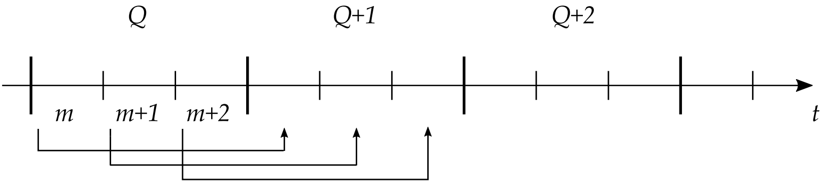

Forecast error

$$u_{t,t+h} = r_{t+h} - \widetilde{\mathbb{E}}_t r_{t+h}$$

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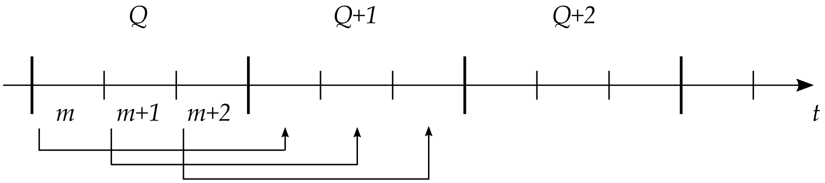

Forecast error

$$u_{t,t+h} = r_{t+h} - \widetilde{\mathbb{E}}_t r_{t+h}$$

H_0: $u_{t,t+h}$ is uncorrelated with QE_{t-1}

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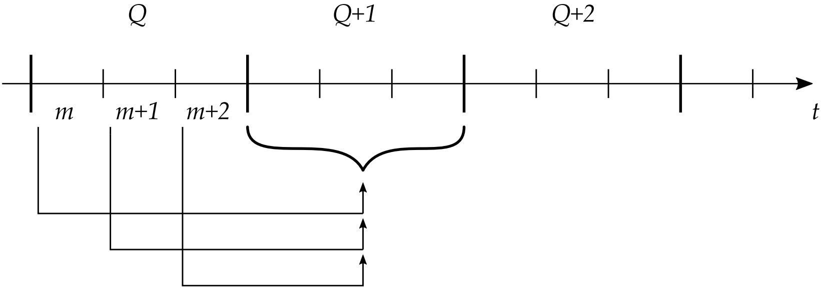
Forecast Errors

Blue Chip Financial Forecasts (BCFF)



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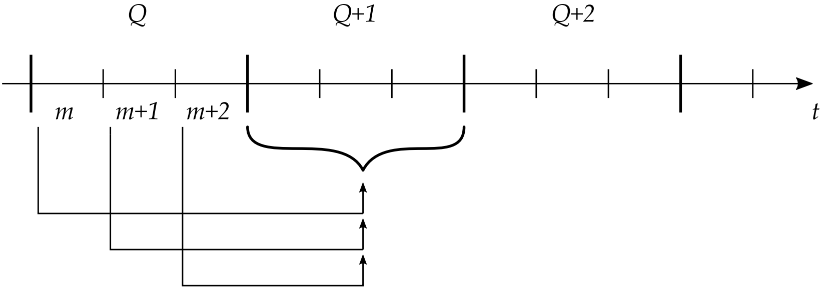

BCFF next calendar quarter forecast error

$$\widetilde{u}_{t,t+\text{“}1:3\text{”}} = \frac{\sum_{i=0}^{2} r_{t+3+i-mod(t+2,3)}}{3} - f_t^{\text{“}1:3\text{”}}$$

H_0: $\widetilde{u}_{t,t+\text{“}3(k-1)+1:3k\text{”}}$ is uncorrelated with QE_{t-1}

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